Definitions and key facts for section 5.1

An **eigenvector** of an $n \times n$ matrix A is a *nonzero* vector $\mathbf{x}$ such that $A\mathbf{x} = \lambda\mathbf{x}$, for some scalar λ. A scalar λ is called an **eigenvalue** of A if there is a nontrivial solution $\mathbf{x}$ of $A\mathbf{x} = \lambda\mathbf{x}$; such an $\mathbf{x}$ is called an *eigenvector corresponding to* λ.

If λ is an eigenvalue of A, then the set of *all* solutions (including $\mathbf{0}$) to

$$(A - \lambda I)\mathbf{x} = \mathbf{0}$$

is the **eigenspace** of A corresponding to λ.

Fact: The eigenspace of A corresponding to λ is a *subspace* of $\mathbb{R}^n$. Indeed, it is

$$\text{Nul}(A - \lambda I) = \{\mathbf{x} \text{ in } \mathbb{R}^n : (A - \lambda I)\mathbf{x} = \mathbf{0}\}.$$

Facts about eigenvalues:

1. The eigenvalues of a triangular matrix are the entries on its main diagonal.
2. If $\mathbf{v}_1, \ldots, \mathbf{v}_r$ are eigenvectors that correspond to distinct eigenvalues $\lambda_1, \ldots, \lambda_r$ of an $n \times n$ matrix A, then the set $\{\mathbf{v}_1, \ldots, \mathbf{v}_r\}$ is linearly independent.